Logarithmic Functions

The inverse of $f(x) = b^x$ is called a logarithmic function with base $b$ and is denoted $\log_b x$

This means that if $f(x) = b^x$ and $b > 0$ and $b \neq 1$ then $f^{-1}(x) = \log_b x$

sooo..... from this we can derive:

<table>
<thead>
<tr>
<th>Logarithmic Form</th>
<th>$\log_b x = y$</th>
<th>if and only if</th>
<th>Exponential Form</th>
<th>$b^y = x$</th>
</tr>
</thead>
</table>

Basic Properties of Logarithms:
If $b > 0$, $b \neq 1$, and $x$ is a real number, then the following statements are true

1. $\log_b 1 = 0$
2. $\log_b b = 1$
3. $\log_b b^x = x$
4. $b^{\log_b x} = x$, $x > 0$

Evaluate each logarithm.

1.) $\log_3 81$
2.) $\log_5 \sqrt{5}$
3.) $\log_{\frac{1}{9}} (1/49)$
4.) $\log_2 2$
5.) $\log_5 512$
6.) $\log_4 4^{1.2}$
7.) $\log_{\frac{1}{32}} (1/32)$
8.) $\log_3 \frac{1}{27}$
9.) $\log_{17} 17$
10.) $2^{\log_{22} 15.2}$
11.) $\log_{16} \sqrt{2}$
12.) $\log_8 \frac{1}{64}$

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<th>Exponential Form</th>
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<tr>
<td>$\log_3 81$</td>
<td>$3^y = 81$</td>
</tr>
<tr>
<td>$\log_5 \sqrt{5}$</td>
<td>$5^y = \sqrt{5}$</td>
</tr>
<tr>
<td>$\log_{\frac{1}{9}} (1/49)$</td>
<td>$(\frac{1}{49})^y = 7$</td>
</tr>
<tr>
<td>$\log_2 2$</td>
<td>$2^y = 2$</td>
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<td>$\log_5 512$</td>
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<td>$\log_8 \frac{1}{64}$</td>
<td>$8^y = \frac{1}{64}$</td>
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Common Logarithm: a log of base 10
the properties of logs also apply to common logs. (You can use your calculator to find
common logs of positive real numbers)

Evaluate:
1.) \( \log 26 \approx 1.415 \)  
2.) \( \log .001 = -3 \)  
3.) \( 10^{\log 5} = 5 \)

4.) \( \log(-5) = \text{no real solutions b/c 5 is negative} \)  
5.) \( \log 10,000 = 4 \)

6.) \( \log .081 \approx -1.092 \)  
7.) \( \log -0 = \text{no real solutions} \)  
8.) \( 10^{\log 3} = 3 \)

A logarithm with base \( e \) or \( \log_e \) is called a natural logarithm and is written \( \ln \).

The natural logarithmic function \( y = \ln x \) is the inverse of the exponential function \( y = e^x \).

Basic Properties for Natural Logarithms:
1. \( \ln 1 = 0 \)
2. \( \ln e = 1 \)
3. \( \ln e^x = x \)
4. \( e^{\ln x} = x, x>0 \) (Approximations of natural logarithms of positive real numbers
   can be found in a calculator by using the ln key).

Evaluate:
1.) \( \ln e^{.73} = .73 \)  
2.) \( \ln(-5) = \text{undefined} \)  
3.) \( \ln 4 \approx 1.386 \)

4.) \( e^{\ln 6} = 6 \)  
5.) \( \ln e^{4.6} = 4.6 \)  
6.) \( \ln(-1.2) = \text{no real solution} \)

7.) \( e^{\ln 4} = 4 \)  
8.) \( \ln 7 = 1.946 \)  
9.) \( \ln 32 = 5 \)

10.) \( e^{\ln 4} = 4 \)  
11.) \( \ln \left(\frac{1}{e^3}\right) = -3 \)  
12.) \( -\ln 9 \approx -2.197 \)
When using your graphing calculator to graph logarithms you must type in the log and x value and then divide it by log of your base number.

Ex \( \log_3 5x \) --- to graph go to y= and type in \( \log(5x)/\log(3) \)

Translations:
- negative outside leading coefficient: reflects across x-axis
- negative inside (): reflects across y-axis
- Add # inside (): Translates left
- Add # outside (): Translates up
- Subtract # inside (): Translates right
- Subtract # outside (): Translates down
- 0<#<1 leading coefficient outside (): expands horizontally
- #>1 leading coefficient outside (): compresses horizontally
- 0<#<1 leading coefficient inside (): expands horizontally
- #>1 leading coefficient inside (): compresses horizontally
Ex. Use the graph of \( f(x) = \log x \) to describe the transformation that results in each function. Draw the function and name its domain, range, y-intercept, x-intercept, asymptote, and end behavior.

Ex.1) \( k(x) = \log (x + 4) \)

- **Transforms left 4 units**

  - **Domain:** \((-4, \infty)\)
  - **Range:** \((-\infty, \infty)\)
  - **y-intercept:** none
  - **x-intercept:** 0
  - **Asymptote:** \( x = -4 \)
  - **Transformations:** down 5 units and reflects across the x-axis
  - **End Behavior:**
    \[
    \lim_{x \to -4^+} f(x) = -\infty \\
    \lim_{x \to 4^-} f(x) = \infty \\
    \lim_{x \to \infty} f(x) = \infty
    \]

Ex.2) \( m(x) = -\log(x) - 5 \)

- **Domain:** \((0, \infty)\)
- **Range:** \((-\infty, \infty)\)
- **y-intercept:** none
- **x-intercept:** -3
- **Asymptote:** y-axis
- **Transformations:** left 4 units
- **End Behavior:**
  \[
  \lim_{x \to 0^+} f(x) = -\infty \\
  \lim_{x \to \infty} f(x) = \infty
  \]